

SPHERES ON A VIBRATING PLATE: CLUSTERING AND COLLAPSE

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A large number ($\sim 10,000$) of uniform stainless steel balls comprising less than one layer coverage on a vertically shaken plate provides a simple but striking demonstration of some of the unusual phenomena associated with excited granular media. When the oscillation amplitude is large enough, the spheres are in constant motion, colliding randomly, like a two dimensional gas in equilibrium. Inter-particle collisions couple the horizontal motion of the gas to the essentially chaotic vertical motion of the spheres driven by the plate vibration. Viewed from above, the horizontal motion in the layer demonstrates interesting collective behavior as a result of inelastic particle-particle collisions. Clusters appear as localized fluctuations from purely random density distributions, as demonstrated by increased particle correlations. The clusters grow as the medium is "cooled" by reducing the amplitude of the plate vibration. The increase in local dissipation leads to the nucleation of a collapse: a close-packed crystal of particles that come to rest on the shaking plate and with respect to one another. The collapse is a motionless condensate surrounded by the remaining particles that continue in motion as a less dense gas. The two phase co-existence is hysteretic, and can be related to the chaotic trajectories of an individual inelastic sphere on an oscillating plate.

1 Introduction and Background

Granular media, systems comprised of large numbers of macroscopic particles interacting only through contact forces, display a variety of interesting behavior not found in ordinary solids, liquids, and gases.¹ In a static granular system, e.g. a pile of sand, the unusual properties are a consequence of the disorder generated by gravitational barriers to relaxation: the energy required to move one particle over another is many orders of magnitude greater than thermal energies. When sufficient kinetic energy is added to the granular system, the grains are no longer in constant contact with one another and the medium becomes 'fluidized'. The peculiar properties of the static sandpile give way to a state characterized by rapidly fluctuating particle velocities, much like the motion of molecules in a liquid or gas. Because of the dissipation generated by colliding grains, energy must be continually added to the system, for example by external gas flow or mechanical vibration, and the results of equilibrium thermodynamics and statistical mechanics are not directly applicable. The unusual properties of granular gases have been investigated primarily through theoretical analyses and numerical simulation of freely cooling granular media.² A collection of grains is initialized with a random velocity distribution (usually a Boltzmann distribution), and random initial positions. As energy leaves the system through dissipative inter-particle collisions, the homogenous state becomes unstable to the formation of large scale inhomogeneities in density and velocity. This instability is a consequence of the dissipation: if the density of some portion of the gas increases due to a random fluctuation, the collision rate will increase, resulting in an increase in the dissipation rate. This local 'cooling' of the granular gas will decrease the effective pressure, allowing for a further influx of particles from the surrounding gas.

Under some circumstances this instability can lead to inelastic collapse, where the particles essentially come to rest next to one another in the center of mass frame.

Many researchers in the nonlinear dynamics community have become interested in fluidized granular media because of the close relationship between granular dynamics and the dynamics of other nonlinear, dissipative systems. Granular fluids display a variety of instabilities similar to those seen in fluids,¹ but the underlying equations of motion are unknown. In addition, the methods used to input energy into the system are themselves interesting dynamical systems. In gas-fluidized granular media, energy is input by flowing a gas through the grains. The instabilities associated with the flow past an isolated sphere, together with the interactions between the spheres, combine to produce the complex dynamics of gas-fluidized beds. For mechanical vibration, the energy is input through collisions between the grains and the vibrating boundary. The interesting dynamics of this simple system is discussed in section 2.1. We have been investigating the behavior of a single layer composed of a large number of spherical particles on a vibrating plate. Some of our results, reviewed below, have been previously reported.^{3,4}

2 Experimental apparatus

A schematic of the experimental apparatus is shown in Fig. 1. A 20 cm diameter horizontal aluminum plate with a smooth surface is vibrated vertically by an electromagnetic shaker. The particles placed on the surface are typically 1 mm stainless steel balls with a uniformity of better than 0.5%, and are contained by a wall around the

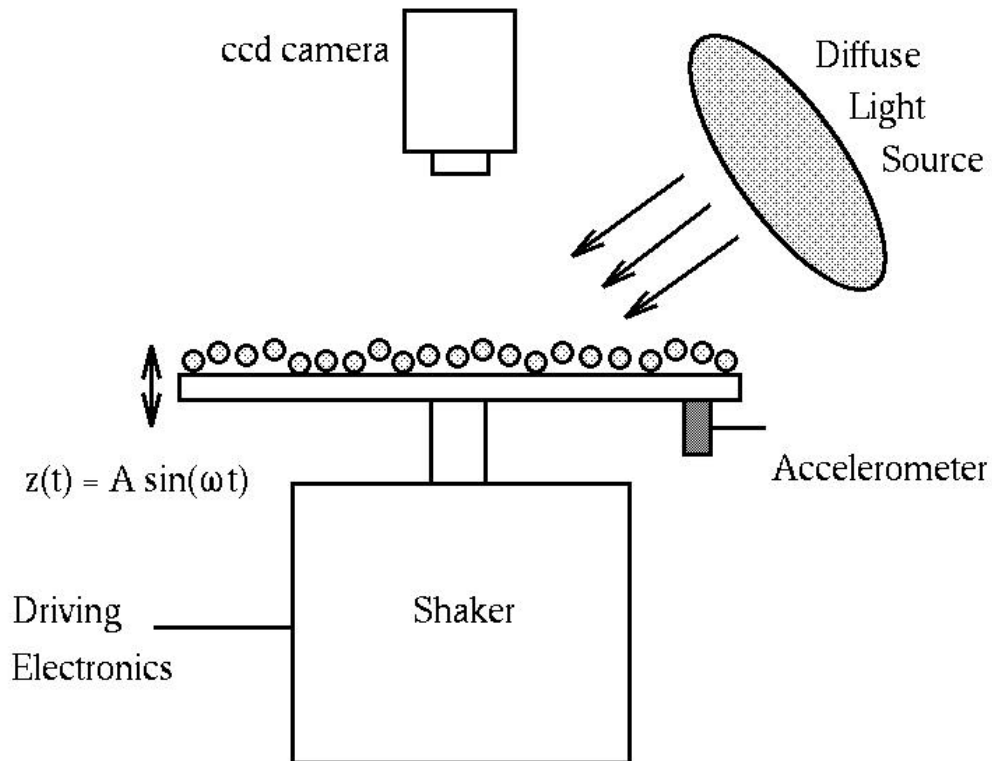


Fig 1. A schematic of the experimental apparatus.

outside of the plate. The coverage is always less than one full layer. The acceleration of the plate is monitored with a fast-response accelerometer mounted on the bottom surface of the plate. The particles are illuminated by low angle diffuse light. This illumination produces a small bright spot at the top of each particle when viewed through a video camera mounted directly above the plate. Two digital video cameras are used for data acquisition, a high-resolution camera for studying spatial correlations (Pulnix-1040, 1024 x 1024 pixels, Pulnix America, Inc., Sunnyvale, CA), and a high-speed camera for measuring velocity distributions (Dalsa CAD1, 128 x 128 pixels, 838 frames/second, Dalsa Inc., Waterloo, ON; Canada).

2.1 Relationship to a single particle on a vibrating plate

In the experimental setup we have easy access to the horizontal positions and velocities of the particles, but cannot directly measure what is happening in the vertical direction. It is nonetheless clear that the vibration of the plate inputs energy into the vertical kinetic energy of the particles, and this energy is transferred to the horizontal components through inter-particle collisions. Neither process is quantitatively understood.

In thicker layers, the kinetic energy of the granular media is dissipated rapidly, and the interaction of the layer with the plate can be reasonably well modeled by a completely inelastic ball.⁵ For sinusoidal oscillations, the ball remains at rest on the plate if the peak plate acceleration is less than $1g$, the acceleration due to gravity. If the amplitude of the acceleration is increased a little above $1g$, the ball will separate from the plate at that point in the cycle where the acceleration reaches $-g$, and then recollide with plate and lose all relative velocity at some point after the plate acceleration has increased above $-g$ again. At higher accelerations, period doubling bifurcations and other interesting dynamical phenomena are observed.⁶

The coefficient of restitution between the stainless steel balls and the aluminum plate in our experiment is approximately 0.9, so the individual particle-plate collisions are closer to completely elastic than completely inelastic, and because our coverage is always less than one full layer, inter-particle collisions do not dissipate energy very rapidly. As a consequence the particles collide with the plate randomly throughout the oscillation cycle. This can be seen directly in Fig. 2, which shows the signal recorded by the accelerometer attached to the plate. The dominant low frequency response is due to the sinusoidal forcing, but each time a particle collides with the plate, a high frequency impulse is generated as well. Measuring the *rms* amplitude of the high frequency component of the signal provides a measure of the relative probability of plate collisions through the cycle. As expected, collisions occur more frequently when the plate is moving upward, but no more specific correlation is observed.

The dynamics of a partially elastic sphere on a vibrating plate have been extensively studied⁷, and displays many features typically associated with chaotic dynamical systems. One relatively unusual feature is the appearance of locking, where the ball will come to rest on the plate by 'chattering' down during a period where the plate acceleration is above $-g$.⁸ When the plate acceleration drops below $-g$ again, the ball will separate from the plate. Since most chaotic trajectories will eventually diffuse into a region of phase

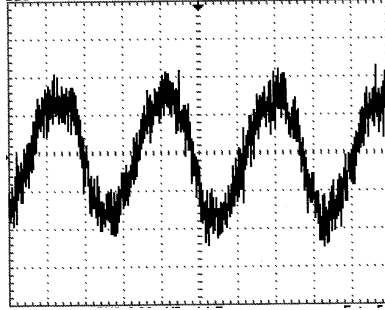


Fig. 2 Signal from accelerometer for a plate oscillation of 70 Hz and $\Gamma=1$, showing sinusoidal forcing as well as the high frequency pulses generated by particle-plate collisions. The grid spacing is 5 milliseconds on the horizontal axis, and approximately $1/2 g$ on the vertical axis.

space that leads to chattering, the generic asymptotic trajectories are in fact periodic, although the periods of the trajectories can be extremely long. If the peak plate acceleration is less than g , a ball that chatters down will never again leave the plate, remaining in a trivial 'ground state' attractor. Thus initial conditions will typically lead either to relatively simple periodic trajectories or to the ground state attractor. Figure 3 shows two example trajectories for a point particle calculated from the equations of motion, using a coefficient of restitution of 0.9, and a peak acceleration of $0.95g$. The initial conditions were chosen to demonstrate two characteristic behaviors: the ball in the top panel quickly chatters down to rest on the plate, while the ball in the bottom panel, dropped from a slightly lower initial height, is approaching a period 1 orbit.

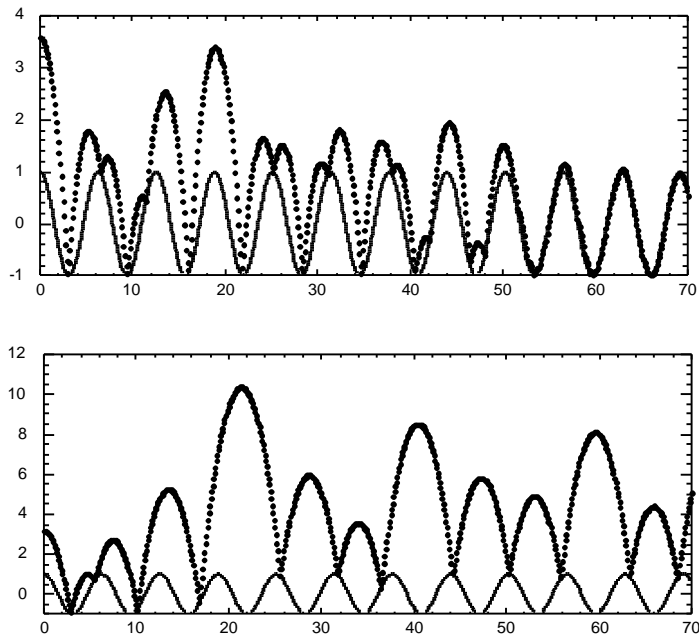


Figure 3. Two trajectories for a point particle on an oscillating plate, for $\Gamma=0.95$

A more complete picture of the dynamics can be seen by taking a phase portrait of a number of trajectories. Figure 4 shows the result for 200 particles, followed for 100 collisions with the plate. At each collision, the phase of the plate and the kinetic energy of the particle are recorded. The dense clouds of point around an energy of 5.5 come from trajectories approaching the period-one orbit such as the bottom panel of Fig. 3, and the cluster of points close to the x-axis comes from trajectories chattering down to rest on the plate. Losert, et al⁹ have noted that there are apparently no periodic orbits with energies below that required for a period-one orbit (where the velocity must be such that the flight time equals one oscillation period). This 'energy gap' has important consequences for the granular layer.

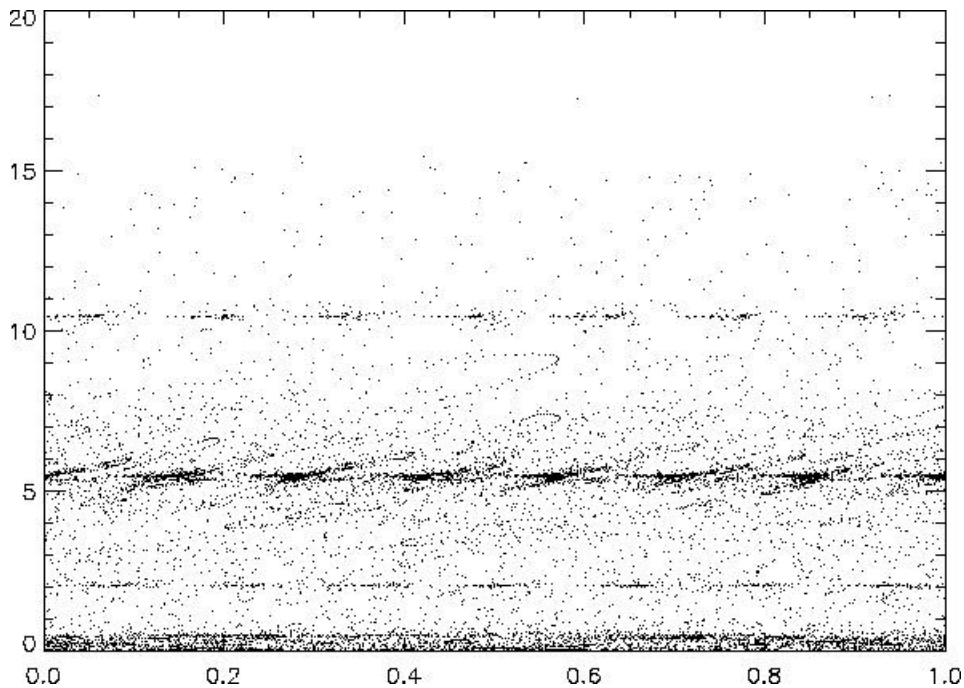


Figure 4. Phase portrait of 200 trajectories followed for 100 collisions with the plate. At each collision, the phase of the plate oscillation divided by 2π (x-axis) and the kinetic energy of the ball (y-axis) are recorded.

In principle, a collection of perfectly spherical particles on a flat plate vibrating vertically with no initial horizontal velocities would not interact. Experimentally, this is not observed for any reasonable plate coverage. If the plate acceleration is increased from zero, rapid, random horizontal motion sets in almost immediately after the particles pick up significant vertical kinetic energy. This instability has been studied experimentally in a system similar to that shown in Fig. 1.⁹

3 Results

When the plate oscillation amplitude is not too large, the spheres never hop over one another; thus the system essentially two-dimensional. Nonetheless, there is sufficient energy in the horizontal velocity component to generate fascinating dynamic phenomena.

At moderately large sinusoidal vibration amplitudes (maximum plate acceleration slightly larger than g), a fully fluidized state is observed. The spheres are constantly in motion and there is no large-scale spatial ordering. Figure 5(a) shows an instantaneous image of part of a cell containing 8000 particles (just under half a monolayer) in this regime. This phase is characterized by an apparently random distribution of particle positions and velocities. A sense of the dynamics can be gained from an average of 15 frames taken over a period of 1 s (Fig. 5(b)), which shows the lack of any stable structure. As the amplitude of the acceleration is slowly decreased, the average kinetic energy of the particles decreases, and localized transient clusters of low velocity particles appear.

An instantaneous image in this regime (Fig. 5(c)) does not look very different from the one taken at higher acceleration, but in the time-averaged image (Fig. 5(d)) bright peaks are clearly evident, corresponding to low-velocity particles that have remained relatively close to each other over the time interval. In this regime, the clusters typically survive for 1-20 seconds. In the low density regions outside of the clusters, there are

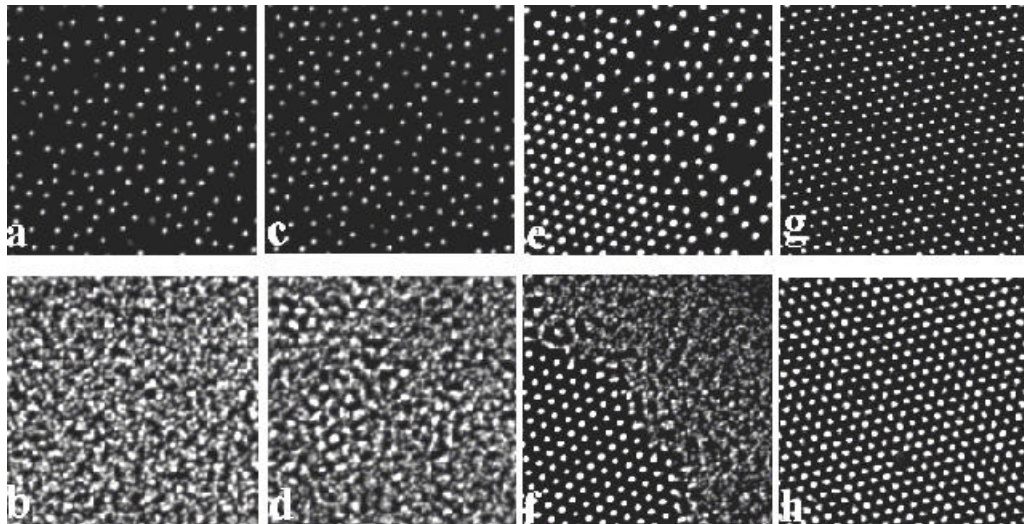


Figure 5. Instantaneous (top row) and 1 second time-averaged (bottom row) photographs detailing the different phases of the granular monolayer. (a), (b), uniform particle distributions typical of the gas phase. (c) The clustered phase. The higher intensity points in a time-averaged image, (d), denote slower, densely packed particles. (e) A portion of a collapse. (f) The time-averaged image shows that the particles in the collapse are stationary while the surrounding gas particles continue to move. (g),(h) In a more dense system at higher frequency, there is an ordered phase where all of the particles remain in motion. (After ref. 3)

particles with anomalously high velocities, and these appear to be responsible for the breakup of the clusters. There are no attractive interactions between these particles; the cluster formation is a uniquely non-equilibrium phenomenon, resulting from the dissipation during inter-particle collisions.

When the amplitude of the vibration is decreased somewhat below that of Fig. 5(c,d), the typical cluster size increases to 12-15 particles. Within a few minutes at this acceleration, one of these large clusters will become a nucleation point for a 'solid' phase,

similar to what is referred to as 'inelastic collapse'.² The particles in the collapse are in contact with all of their neighbors, and form a perfect hexagonal lattice (Fig. 5(d)). The collapse is surrounded by a gas of the remaining particles. The sharp interface between the coexisting phases can be seen in the time-averaged image (left panel of Fig. 5(e)). The two-phase co-existence persists essentially unchanged for as long as the driving is maintained.

The collapse is presumably nucleated by density fluctuations generated by the random motion of the particles in the gas. The behavior is analogous to first order transitions in equilibrium systems, when the high temperature phase is quenched to some temperature below the critical temperature for nucleation of the low temperature phase. The deeper the quench, the quicker the nucleation. The granular gas can be rapidly 'quenched' by abruptly decreasing the amplitude of the acceleration. The granular temperature drops almost immediately, because the inter-particle collisions rapidly dissipate the excess kinetic energy. Figure 6 shows the time required for the collapse to formed and grow to about its steady-state size after the plate acceleration is abruptly reduced from 1.15g. If the final acceleration is below about 0.88g, several nucleation sites typically form almost immediately, and the resulting collapse is polycrystalline. For quenches closer to the critical acceleration, a single nucleation site typically forms, but the time required for nucleation grows rapidly as the depth of the quench is reduced. As expected for a process dominated by fluctuations, the variation in the nucleation time

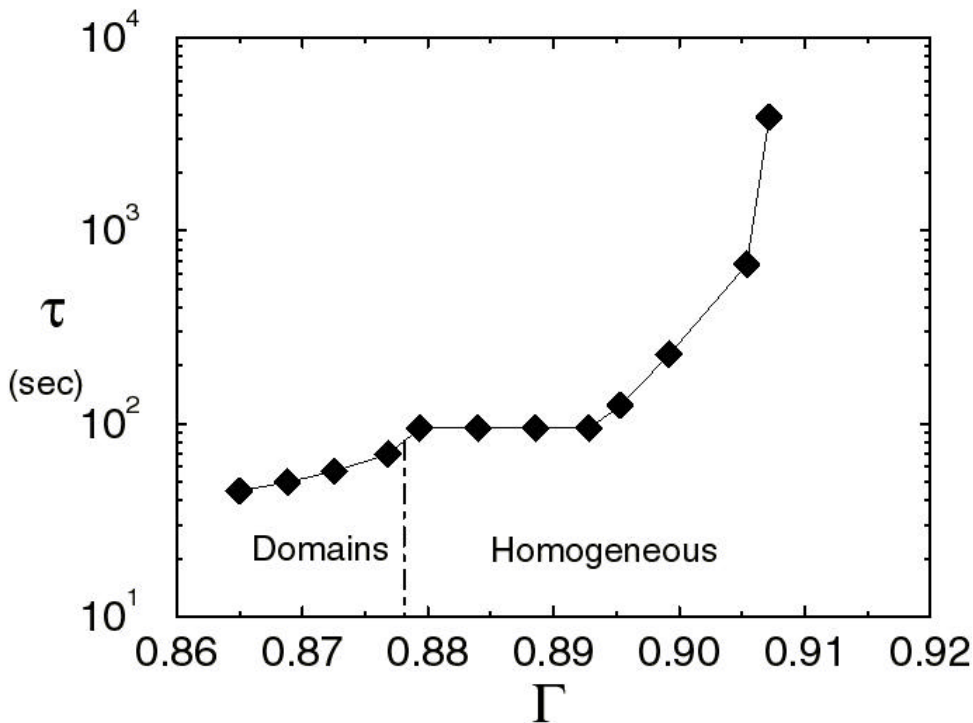


Figure 6 Time until the formation of a collapse after an abrupt decrease in the acceleration from $\Gamma=1.15$. In the homogeneous regime, a single collapse typically nucleates, while for lower accelerations there are several nucleation points, and the resulting collapse is polycrystalline.

from one run to the next can be quite large, and a more careful investigation is required before the critical behavior can be quantitatively investigated.

At higher densities, instead of a transition directly from the clustering behavior to collapse, there is an intermediate phase with apparent long range order. Figure 5(g) shows a monolayer in this ordered state, where the spheres are arranged in a hexagonal lattice but are not at rest or in contact with one another. The disorder in the image is a consequence of the fluctuations induced by inter-particle collisions. When the particle positions in the ordered phase are averaged over a short time (Fig. 5(h)), the resulting image displays a nearly perfect lattice. Measurements of the correlation functions for positional and orientational order parameters in this phase suggest that the transition to this ordered phase is quantitatively similar to the solid-liquid transitions observed in a variety of equilibrium systems. The ordered phase shows long range orientational order, but algebraically decaying positional order. At slightly higher accelerations, the liquid phase shows algebraically decaying orientational order, but only short range positional correlations; in other words, it appears to be a hexatic liquid predicted by the work of Kosterlitz, Thouless, Halperin, Nelson, and Young.¹⁰ It is remarkable that this strongly dissipative, far from equilibrium system apparently displays the subtle dynamics of equilibrium two-dimensional melting.

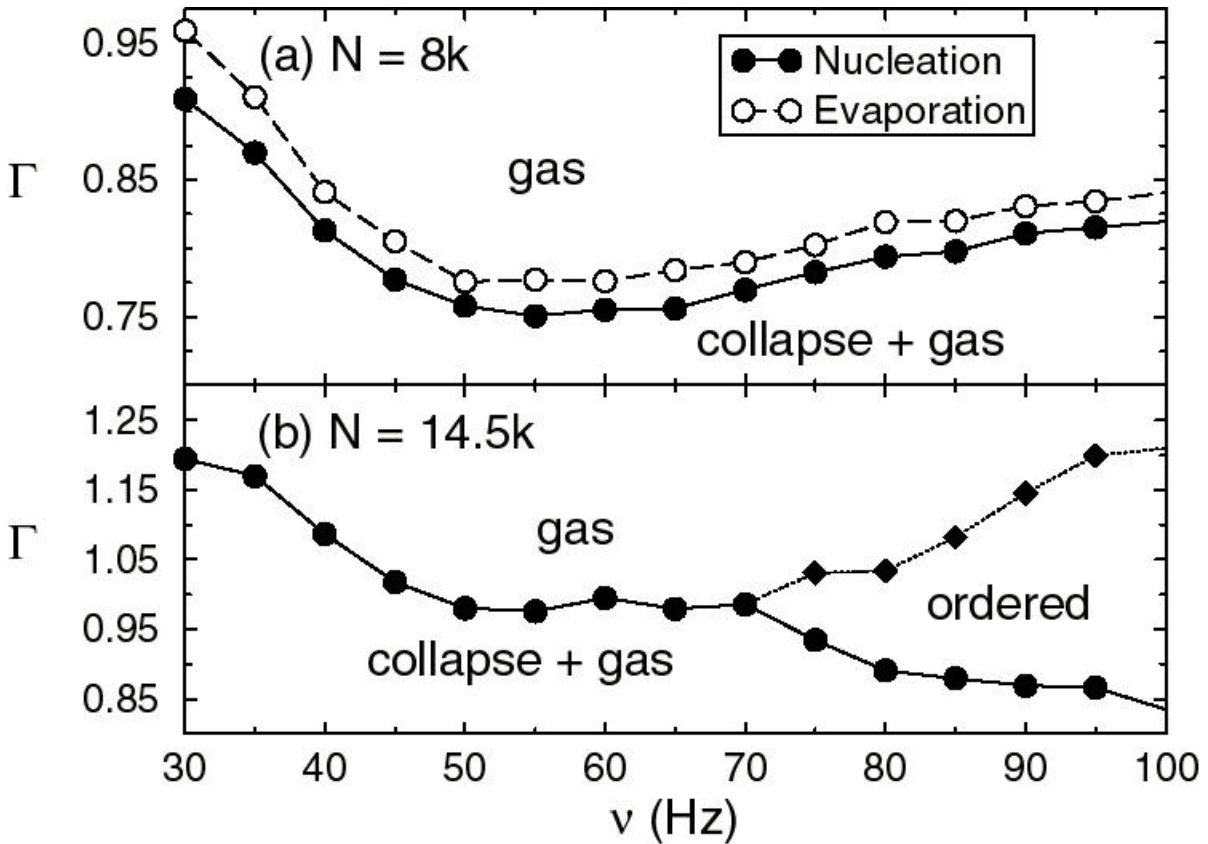


Figure 7. The phase diagram for (a) $N=8000$ and (b) $N=14,500$ particles. The filled circles indicate the acceleration where the collapse nucleates (see text). The open circles in (a) indicate the point where the collapse disappears when the acceleration is increased. The diamonds in (b) indicate approximately where the transition to the ordered state occurs. (After ref. 3)

Figure 7 shows phase diagrams of the system with two different densities. The hysteresis is omitted from the lower diagram for clarity. The data for the nucleation points were taken by decreasing the plate acceleration in steps of about 0.003g, and waiting 5 minutes at each step to see if the collapse nucleates. From Fig. 6, it is clear that the precise location of the nucleation line will depend on the waiting time, but only very weakly. It is not immediately clear what causes the frequency dependence in the phase diagram. For an ideal spherical particle on an oscillating plate with a velocity-independent coefficient of restitution, the dynamical behavior depends only on Γ , and the frequency sets the timescale for the motion, and the length scale through g/ω^2 , which is proportional to the distance a ball falls during one oscillation cycle. Thus as the frequency is reduced for fixed Γ , balls will bounce higher. Because the balls in this system interact with their neighbors, it is possible that the frequency dependence enters through the ratio of this length scale to the ball diameter. In fact the rapid increase in the acceleration where collapse forms for frequencies below about 50 Hz (see Fig. 7) occurs when the particles begin to bounce high enough to hop over one another, resulting gradual transition from primarily 2D to 3D dynamics. This suggests that the frequency dependence comes from a characteristic frequency $v_c=(g/d)^{1/2}$, where d is the sphere diameter. Figure 8 shows the phase diagram measured for two different sets of particles, one with a diameter of 1.2 mm, and one with a diameter of 1.6 mm. The upturn at low frequencies and the appearance of the ordered phase occur at lower frequencies for the larger particles, but the scaled phase diagrams lie right on top of one another.

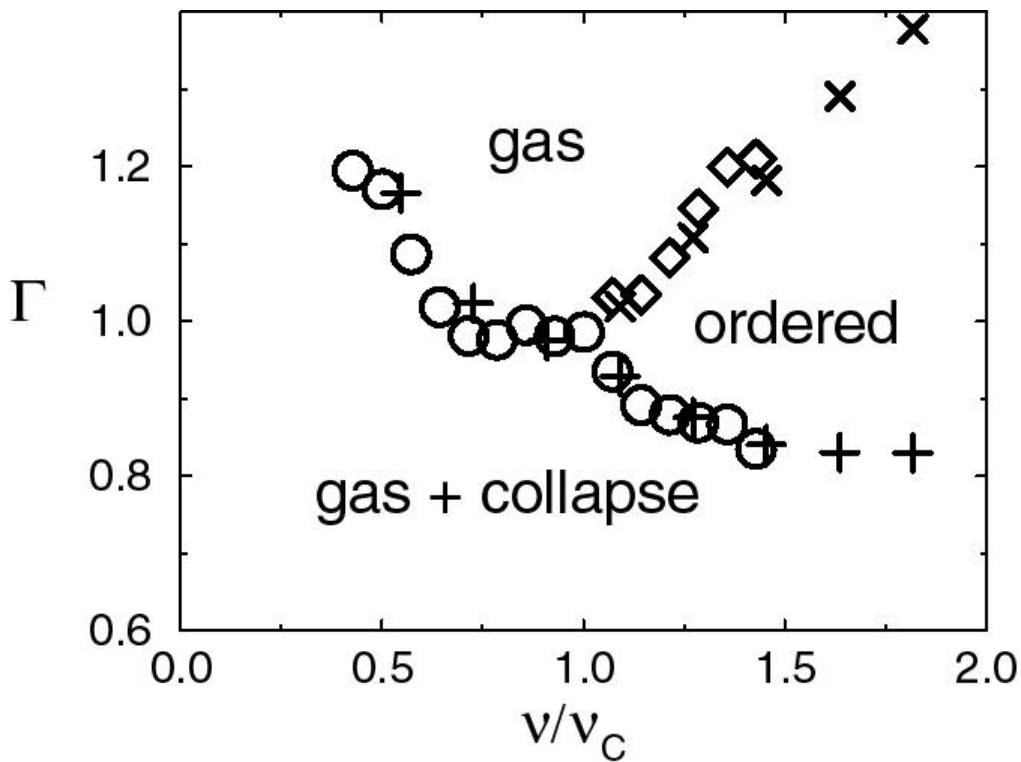


Figure 8. The phase diagrams for 1.2 mm diameter particles (o, ?) and 1.6 mm (+, x). The frequency is scaled by $v_c=(g/d)^{1/2}$.

3.1 Non-Maxwellian velocity distributions in the gas phase

As described above, the clustering is characterized by a decrease in the relative velocities of particles in the cluster. Dissipative particle systems will not necessarily show a Maxwellian velocity distribution, although models often assume that form for lack of anything better to use, and by arguing that it should be appropriate in the limit of low dissipation. The velocity distribution has been experimentally measured in a few cases, but not with sufficient precision to resolve the functional form, in part because these systems were inhomogeneously forced, with the energy input coming from the sides or from the bottom of a thick granular layer. The translational invariance of the system we are investigating facilitates precise measurements of statistical quantities because the entire system (sufficiently far from the walls) can be included in the ensemble.

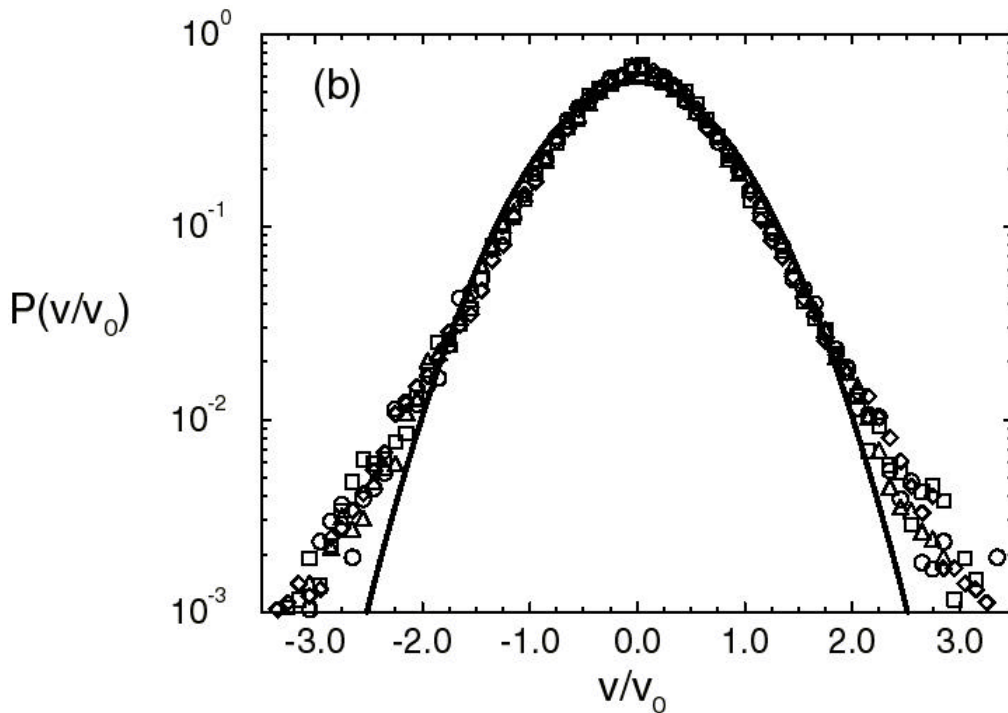


Figure 9. Probability distribution function for a single component of the horizontal velocity. The data represents several different experimental parameters for values of Γ from 0.76 to 1.0. Each distribution is scaled by $v_0 = (2v_{2m}^2)^{1/2}$, where v_{2m} is the second moment of the distribution. (After ref. 3)

With the use of a high speed digital video camera, the particle velocities between collisions can be determined in the granular monolayer. Our measurements of the velocity distributions in the plane of the granular gas clearly show non-Gaussian behavior. In Fig. 9 the experimentally measured velocity probability distributions for several different parameters are shown on a semi-log plot, and compared to a Gaussian distribution (solid line). The distributions are scaled only with a characteristic velocity proportional to the second moment of the distribution. As no asymmetry is observed, the distributions contain both v_x and v_y data. The straight tails in Fig. 9 suggest an exponential velocity distribution.

The deviations from the Maxwell velocity distribution can be understood as clustering in momentum space: when the clusters form in real space they generate a population of low-velocity particles in a the high-density region. The corresponding areas of low density result in regions of decreased dissipation that allow for high-velocity particles. Thus, as seen in Fig. 9, the populations of low- and high-speed particles are higher than that expected from a Maxwell distribution, and the intermediate velocity population is smaller. The behavior of the velocity distributions at higher accelerations and an investigation of the cross-correlations between density and energy fluctuations is reported in reference 4, and non-Gaussian velocity distributions have been reported in a similar system.¹¹

4 Discussion

An understanding of the interesting dynamics displayed in both the gas phase and the two-phase coexistence regions will require a better understanding of the flow of energy from the plate to the granular layer. Energy flow into thicker layers from a vibrating surface have been extensively studied,¹² but the dynamics of the monolayer system are quite different. The net energy transferred to the vertical motion by the plate must balance the energy dissipated by inter-particle collisions. It is this balance that determines the steady-state horizontal granular temperature. The energy dissipated by a two-dimensional inelastic gas can be calculated using kinetic theory.¹³ There are several assumptions that go into the theory that have not been tested experimentally, but numerical simulations suggest that the equations provide a reasonably good model, provided the density is not too large.¹⁴ In order to generate an equation of state from kinetic theory, however, an expression for the energy input by the plate is required. The motion of a single ball on a plate displays the characteristics of low-dimensional chaos, but when coupled with a large number of similar systems through inter-particle collisions, the result is apparently a very regular rate of energy flow, producing a system that looks in many ways very much like an equilibrium system of a large number of interacting particles.

The interaction of the spheres with the plate, and in particular the apparent lack of any periodic or chaotic orbits with average energies less than the period-one orbit, may partially explain the two-phase coexistence and hysteresis observed in this system.⁹ If the kinetic energy of the spheres drops too low, they will fall into the 'ground state', where they remain at rest on the plate, and the energy input drops to zero. Collisions from neighboring spheres may keep a sphere from falling into the ground state, but also dissipate energy. Quantitative predictions of the conditions under which density fluctuations can nucleate a collapse, as well as the stability of the collapse-gas interface, may be derivable from considerations of the sphere-plate dynamics, coupled with the kinetic theory of dense, inelastic gases.

5 Acknowledgements

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